


www.csiro.au

Analysis of a resistive screen

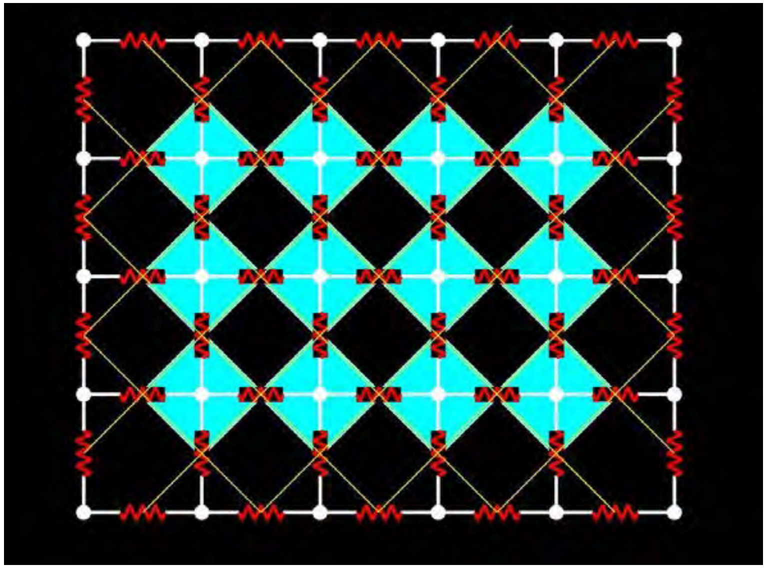
John O'Sullivan, Centre for Astronomy and Space Science, CSIRO

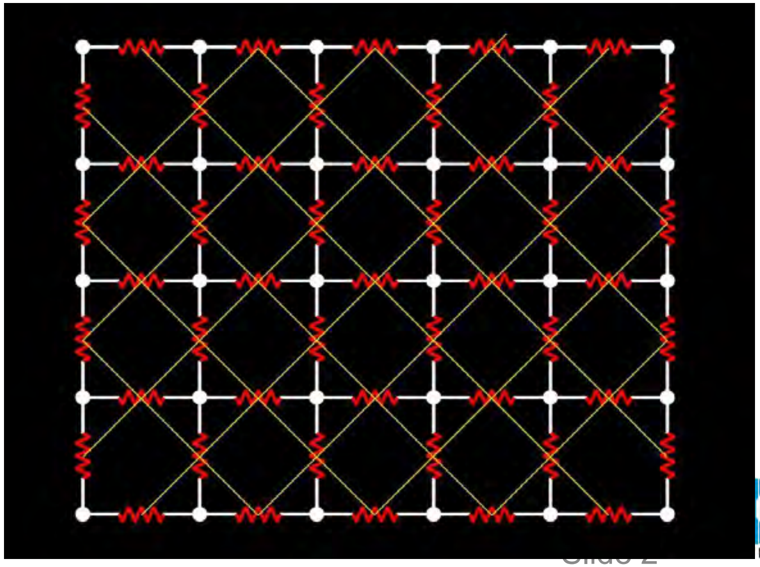
15 November 2012

CSIRO

Background

Want instrumental properties
fundamentally known and stable





Phased arrays – the early days

Antenna Made of an Infinite Current Sheet

HAROLD A. WHEELER, FELLOW, IEEE

Abstract—The simplest concept of a phased array is an infinite planar current sheet backed by a reflecting boundary. The electric current sheet, or resistance sheet, is the limiting case of many small electric dipoles, closely spaced, and backed by an open-circuit boundary. If this array is viewed as a receiver, a plane wave incident on the array at some angle (θ) meets a boundary resistance varying in proportion to cos θ for angles in the H plane, and 1/cos θ for angles in the E plane. If the array is matched at broadside ($\theta=0$), the corresponding reflection coefficient has the magnitude $(\tan \frac{1}{2}\theta)^2$.

While the electric current sheet is realizable, the open-circuit boundary is not. However, a magnetic current sheet can be simulated by a conductive sheet with holes utilized as magnetic dipoles, such a sheet providing the backing equivalent to a short-circuit boundary. The latter case is related to the former by electromagnetic inversion or duality. Therefore, an incident plane wave meets a boundary conductance varying in proportion to cos θ for angles in the E plane, and 1/cos θ for angles in the H plane. The predicted behavior is verified qualitatively by tests of such a model with elements of a practical size.

The derivation is based on the principle of dividing the space in front of the array into parallel tubes or waveguides, one for each element cell in the sheet or array. This is one of the principles published by the author in 1948. A related principle enables the simulation of an infinite array by imaging a few elements in the walls of a waveguide. This latter principle is utilized for making tests of the array.

Manuscript received September 28, 1964; revised February 4, 1965. This paper was originally presented at the IEEE Symposium on Antennas and Propagation, September, 1964.

The author is President of Wheeler Labs., Inc., Great Neck, N. Y.

I. Introduction

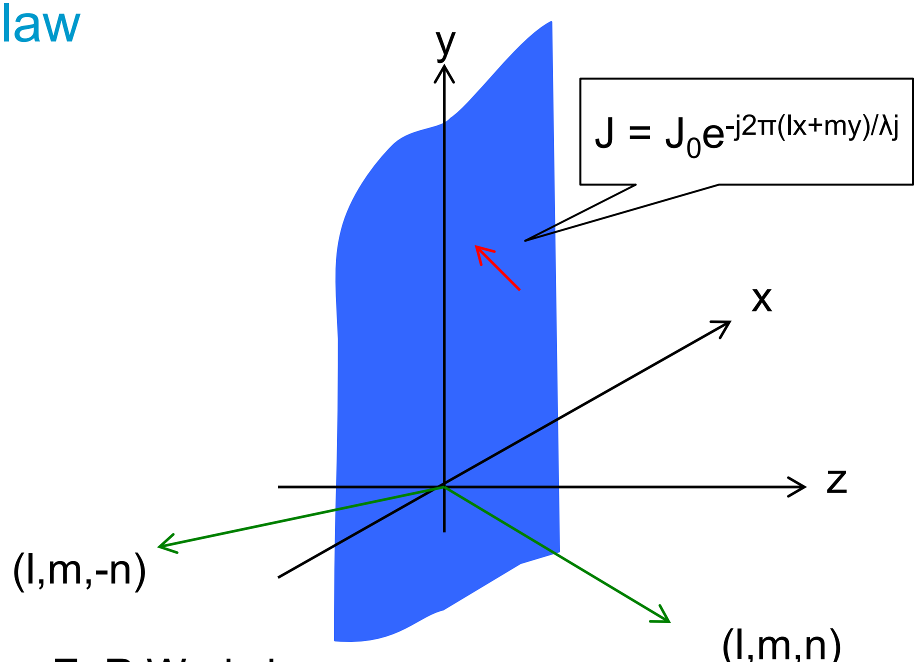
A PLANAR ARRAY of many radiating elements is used to develop a narrow pencil beam. The connections between the elements and the sending or receiving circuit are phased to control the beam angle of deflection from broadside (the "scan angle"). This is the type of antenna commonly called a "phased array," implying beam steering by electrical phasing without mechanical motion of the array.

This type of antenna introduces an unusual problem of design: the variation of element impedance with scan angle. This variation is caused by the inherent coupling of the elements, which contributes to the apparent impedance of each element in a manner dependent on the phasing required for the scan angle. With phasing for wide-angle scanning, the impedance variation from this cause becomes substantial and should be taken into account, along with the variation from other causes such as the frequency bandwidth.

There have been various studies of the element impedance variation with scan angle. Some are based on laborious computation of an array of many elements [5], [7], [12], [13]. Others are based on the concept of an infinite array [2], [6], [8], [14], [17], [18], [21]. This latter concept has been found by the author to offer the best basis for the design of an element for use

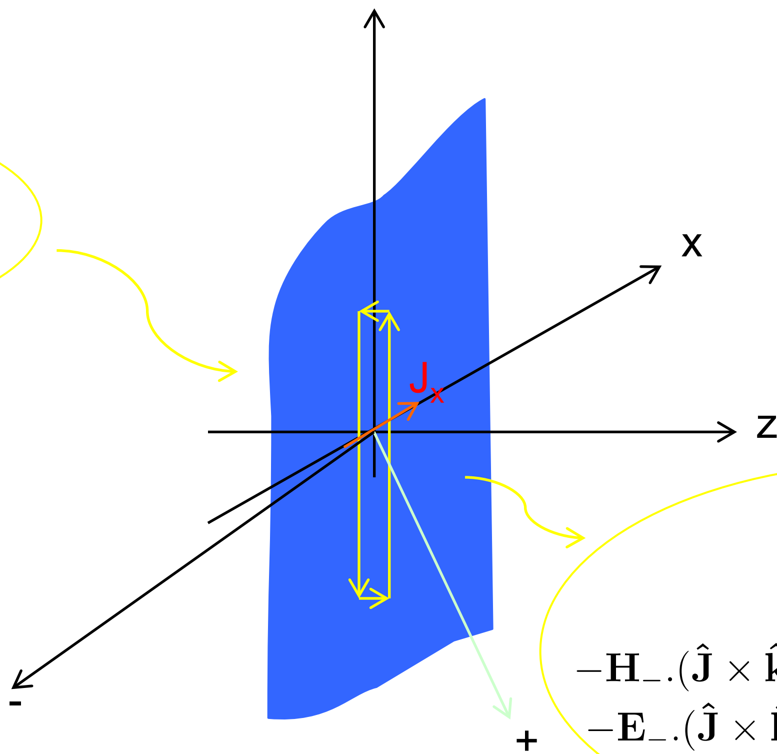
Fundamentals

- Linear phase gradient in 2D current sheet excites/is excited by plane wave radiation
  - Only two plane wave directions consistent
  - Problem invariant of x,y over screen (except for phase)
  - Fourier sum for any other current distribution
- Electromagnetic boundary conditions
- Ohm's law



Boundary conditions

$$\begin{aligned}
\nabla.\mathbf{D} &= \rho \\
\nabla.\mathbf{B} &= 0 \\
\nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}
\end{aligned}$$



Transvers E continuous across sheet.
Transverse H parallel to current continuous.
Normal B continous (not needed)

$$\begin{aligned}
-\mathbf{H}_-.\hat{\mathbf{J}} + \mathbf{H}_+.\hat{\mathbf{J}} &= 0 \\
-\mathbf{E}_-.\hat{\mathbf{J}} + \mathbf{E}_+.\hat{\mathbf{J}} &= 0 \\
-\mathbf{H}_-.(\hat{\mathbf{J}} \times \hat{\mathbf{k}}) + \mathbf{H}_+.(\hat{\mathbf{J}} \times \hat{\mathbf{k}}) &= \mathbf{J}.\hat{\mathbf{J}} \\
-\mathbf{E}_-.(\hat{\mathbf{J}} \times \hat{\mathbf{k}}) + \mathbf{E}_+.(\hat{\mathbf{J}} \times \hat{\mathbf{k}}) &= 0
\end{aligned}$$

Radiation from currents in sheet

- Choose current in x direction and spherical coordinated system with radiated field propagation directions

$$\begin{aligned}\hat{\mathbf{r}}_+ &= [\cos(\theta)\sin(\phi), \sin(\theta)\sin(\phi), \cos(\phi)] \\ \hat{\mathbf{r}}_- &= [\cos(\theta)\sin(\phi), \sin(\theta)\sin(\phi), -\cos(\phi)]\end{aligned}$$

- Then boundary conditions simply give radiated field magnitudes

$$H_0 = -J_0 \frac{\sqrt{\sin(\phi)^2 \sin(\theta)^2 + \cos(\phi)^2}}{2\cos(\phi)} \qquad E_0 = \eta H_0$$

- Radiation resistance given by potential gradient due to fields vs current or by power per unit area in radiated fields

$$\rho_{rad} = \eta \frac{\sin(\phi)^2 \sin(\theta)^2 + \cos(\phi)^2}{2\cos(\phi)}$$

Radiation resistance

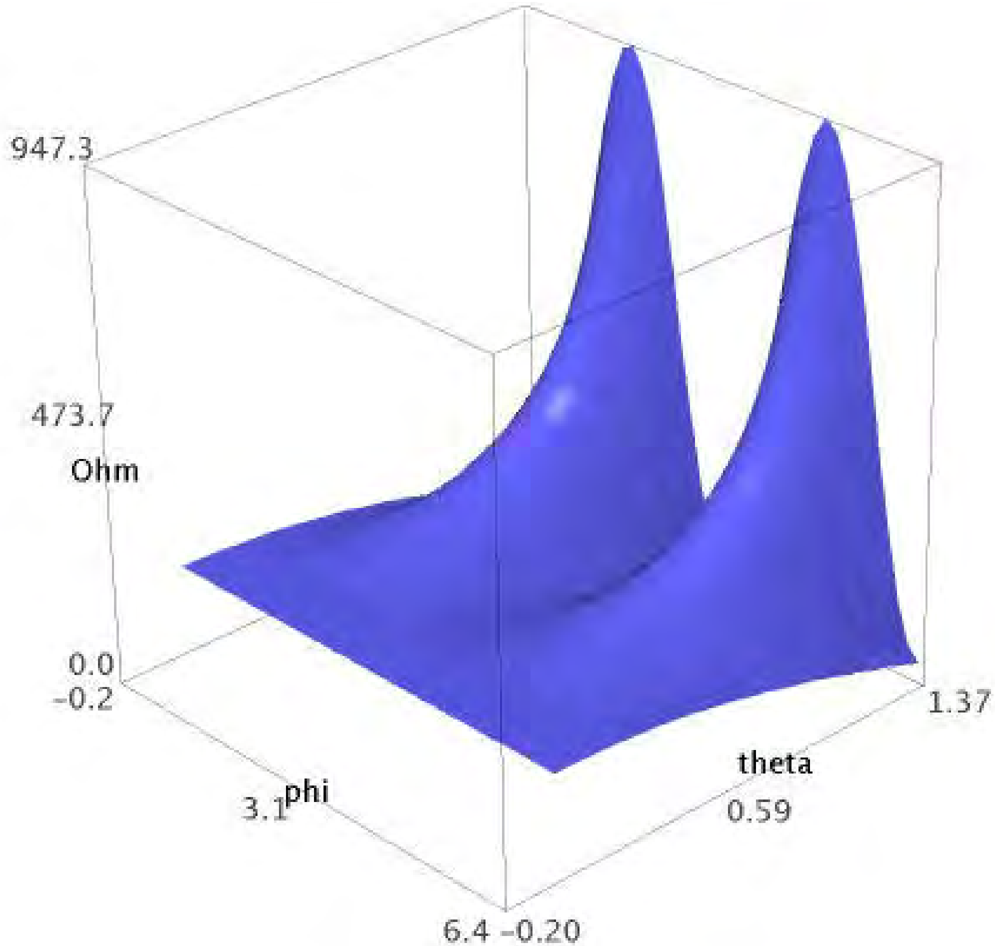
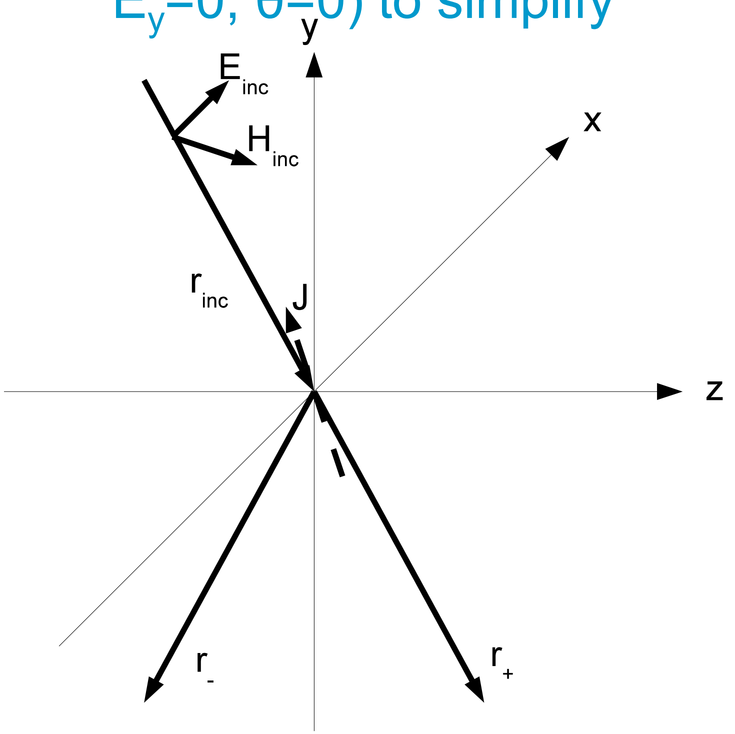


Figure 1: Radiation surface resistivity as a function of (θ, ϕ) for a current direction in the $(1, 0, 0)$ direction

Screen as scatterer

- Incident wave excites current – again + and – scattered waves must have same (l,m) direction components
- General analysis (make easiest use algebra program – Sage)
- Present in terms of vertical and horizontal polarizations (ie E_y=0, θ=0) to simplify



Scattering results

- As expected for sheet resistivity ρ and free space impedance η
  - □⟩ <<| /2 screen acts as reflector
  - □⟩ >>| /2 transmission dominates
- Crossover resistivity different for polarization

$$\begin{aligned} \rho_{cV} &= \frac{\eta\cos(\phi)}{2} \\ \rho_{cH} &= \frac{\eta}{2\cos(\phi)} \end{aligned}$$

- Full results:

$$\begin{aligned} E_{Vm} &= \frac{\eta\cos(\phi)}{\eta\cos(\phi)+2\rho}E_{Vinc} \\ E_{Vp} &= \frac{2\rho}{\eta\cos(\phi)+2\rho}E_{Vinc} \\ E_{Hm} &= \frac{-\eta}{2\rho\cos(\phi)+\eta}E_{Hinc} \\ E_{Hp} &= \frac{-2\rho\cos(\phi)}{2\rho\cos(\phi)+\eta}E_{Hinc} \\ J_V &= \frac{-2\cos(\phi)}{\eta\cos(\phi)+2\rho}E_{Vinc} \\ J_H &= \frac{-2\cos(\phi)}{2\rho\cos(\phi)+\eta}E_{Hinc} \end{aligned}$$

Is this the whole story?

- Not quite!
- Real screen has parasitic inductance and capacitance
- Finite size elements
- Edge effects