The Bizarre World of Special Relativity

Practice Questions for CAASTRO in the Classroom

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The following questions are designed to prepare students in Year 11 and 12 for the lecture on "The Bizarre World of Special Relativity" for CAASTRO in the Classroom. The questions (besides the challenge question) are written for the Year 12 HSC syllabus but Year 11 students should also be able to solve them after introduction of key special relativity concepts, such as Einstein's postulates, time dilation and length contraction. The challenge question is not directly examinable for the HSC but the style and method of solving is common to questions that are examinable.

1. **The Relativistic Cricket Game**

   Anthony and Bill are avid cricket fans and are stuck on a train ride that is travelling at 90% the speed of light. They decide to set up a cricket game in one of the carriages to pass the time. They measure out a distance of a cricket pitch in the carriage to be exactly the standard length of 20.12 m. Bill is not very good at cricket so he ends up just throwing the ball straight at Anthony instead of bowling it. The ball was measured to take 1.2 seconds to reach Anthony.

   Another cricket fan, Julia, is playing cricket on a field as the speedy train passes. She is envious that she is not playing their game on such a fast moving train! So to ensure that Anthony and Bill are meeting the standards of the International Cricket Council, she measures the length of their pitch and how long it takes the ball to leave Bill's hand and to arrive at Anthony's bat.

   (a) Would Julia measure the cricket pitch on the train that is longer or shorter than 20.12 m?

   > **Solution:** Julia is stationary and the pitch on the train is moving relative to her. That means she is trying to work out l_v in the length contraction equation. Since $v < 0$ this means that l_v will be smaller than l_0 as $1/\sqrt{1 - v^2/c^2} > 1$. To make it easy to answer this, remember this phrase - *moving objects are shortened.*

   (b) Would Julia measure the time it takes the cricket ball to reach Anythony to be longer or shorter than 1.2 seconds?

   > **Solution:**
   > Similar to above, Julia is trying to solve for t_v, the time it takes an event to occur in a frame moving relative to her. Therefore, as $1/\sqrt{1 - v^2/c^2} > 1$, this means that the time she measures will be shorter. This can be summed up in the phrase - *moving clocks run slow.*

   (c) What length would Anthony and Bill measure Julia's standard cricket pitch to be? Is this the same as what Julia would measure the length of their pitch to be?

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Solution: To solve this question, it is best to right down the variables you know and what variable you want to solve for. The length of the pitch in its rest frame is $l_0 = 20.2\,m$. The velocity of the frame moving relative to Julia is $v = 0.9c$. We are trying to solve for l_v.

$$l_v = 20.2\sqrt{1-(0.9c)^2/c^2} = 8.8\,\mathrm{m} \tag{1}$$

(d) Which special relativity postulate of Einstein's allows us to compare the two experiments, and why does it allow this comparison?

Solution: While both postulates are vital to the framework of special relativity, the only reason we can compare the two frames is due to the principle of relativity. This is somewhat subtle but the principle of relativity states that the laws of physics is the same in *every* inertial frame of reference. If the laws differed, you would be able to tell an inertial frame of reference from others or make one frame somehow more "correct" than another. Therefore, the way the ball's motion measured in the train and outside the train is consistent.

Hint: You will have to use the time dilation and length contraction equations to solve this problem, as given by

$$l_v = l_0\sqrt{1-\frac{v^2}{c^2}} \text{ and } t_v = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}, \tag{2}$$

where l_v/t_v is the length/time of the object in the moving frame, l_0/t_0 is the length/time of the object in the rest frame, v is the velocity of the frame of interest and c is the speed of light, respectively.

2. **Challenge Question: Jets from a black hole in an X-ray binary**

Most of the known black holes found in the Milky Way have a companion star that orbits around them. A black hole is is a region of spacetime that even light cannot escape, and they are formed from the explosion of a massive star as it reaches the end of its life. As material is accreted onto the black hole from the companion star it often forms a disc and an outflow that can be thought of as a jet (see Figure 1). This system can be thought of the reverse of what you see when you take the plug out of a bath tub, neglecting the added complications of magnetic fields and relativistic particles!

Astronomers observe the light from such a jet has a frequency of 6.67×10^{14} Hz, but they know from the spectral lines that in the frame of reference of the jet the frequency of the light is 5.55×10^{13} Hz. At what speed is the material of the jet moving towards us?

Hint: You may assume that the jet is pointed directly towards us, so you can neglect any directional affects. Additionally, you will need to use the relativistic Doppler effect given by

$$f = \sqrt{\frac{c+v}{c-v}}f_0\,, \tag{3}$$

where f is the observed frequency, f_0 is the rest frame frequency, c is the speed of light and v is the speed of the material in the jet, respectively.

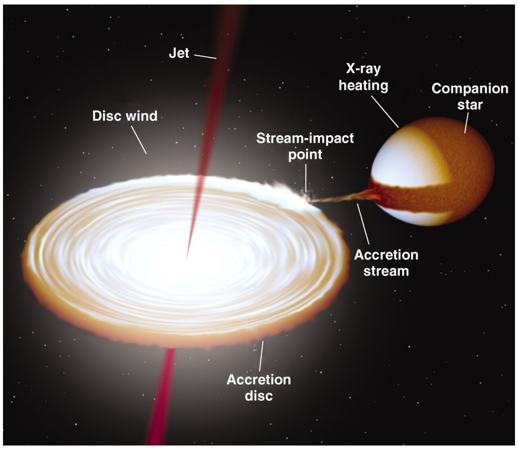

Figure 1: An artist's impression of an X-ray binary. The X-ray emission originates mostly from thermal processes associated with the accretion disc. Image reproduced from Fender & Belloni, *Science* (2012).

Solution:

We are solving for v, this means we must rearrange the equation to be in the form:

$$v = \frac{(f/f_0)^2 - 1}{(f/f_0)^2 + 1}c. \tag{4}$$

We have $f/f_0 = 12.0$, so we find

$$v = \frac{(12.0)^2 - 1}{(12.0)^2 + 1}c = 0.986c. \tag{5}$$

This means the material is streaming out of the black hole system at $\approx$99% the speed of light! The fact this is millions of tons of materials demonstrates the extreme type of environments you get in space and around black holes.